6.2: The Distance to and Age of an Open Cluster

Shoaib Shamsi[1] **and Muhammad Sabieh Anwar**[1]

[1]PhysLab, School of Science and Engineering, Lahore University of Management Sciences.
[*]shoaib.shamsi@lums.edu.pk and sabieh@lums.edu.pk

ABSTRACT

Star clusters can serve as important tools in the study of star formation, dynamics of the interstellar medium, and the evolution of our galaxy. They provide astronomers with stellar populations whose member stars share several properties. These may include astrometry (space velocities, group dispersion), metallicity, and, (especially relevantly to this lab) distance and age. This lab will introduce students to H-R diagrams, stellar clusters, and how different clusters might appear on these H-R diagrams. We will explore how an H-R diagram might allude to the distance and age of a cluster, estimating the distance of the open cluster M35. Additionally, we will use stellar isochrones to estimate the distance, as well as age, of the open cluster NGC 752.

This work has been funded by the National Centre for GIS and Space Applications, Pakistan.
Version December 13, 2022

Learning Objectives

Concepts

Stellar Spectral Classification, Stellar Temperature, Magnitude Scales, Photometric Schemes, H-R Diagram, Stellar Evolution, Open and Globular Clusters, Distance Modulus, Cluster Ages, ZAMS, Isochrones

Skills

Python, pandas, NumPy, AstroPy, Jupyter, Colab

Techniques

Data Analysis, Data Visualisation, Data Filtration, Cluster Photometry, Using Isochrones

1 The Spectral Classification of Stars

In the late 1800s, Harvard College's astronomer Edward Pickering had a lot of analogue spectra—and no easy way to classify them. In these circumstances, he employed one Williamina Fleming, his housekeeper and an expert in creating order out of chaos.

Fleming was a Scottish immigrant who worked for Pickering since her husband abandoned his family. Despite humble beginnings, Fleming quickly rose to become an expert in stellar spectral classification and was one of the founding members of the **Harvard College Computers**. Unfortunately, like many other American scientists, Pickering realised that employing women for highly technical tasks was incredibly beneficial as he could pay them much less. It took several more decades before gender based wage-inequality in the US was addressed (at least legally).

Anyway, Fleming began by classifying spectra based on the strength of the hydrogen absorption lines in the 1880's. Stars with the strongest hydrogen absorption lines are classified as **A** stars, the second strongest were classified as **B** stars, and so on. However, stellar temperature affect the strength of the hydrogen absorption lines, and thus, they alone aren't a great way to classify stars. Later on, another member of the Harvard College Computers, Annie Jump Cannon reclassified the spectra, taking other absorption lines into account. She classified them in order of decreasing temperature, which led to the **OBAFGKM** spectral classification scheme. For for details, check out pages 602–610 from Fraknoi, Morrison, and Wolff (2018).

2 Magnitudes: Apparent and Absolute

Before we go any further, it'll be good to discuss how astronomers talk about the luminosity of stars. We can define the luminosity of stars to be the total energy output of a star across all wavelengths of light. For instance, the Sun outputs around 3.827×10^{26} W, commonly defined to be 1 $L_\odot$.

In around 150 B.C.E. Hipparchus classified stars according to their apparent brightness. Without telescopes or other modern equipment, he estimated the **magnitude** for around 1000 stars to be a number from 1 to 6, with magnitude 1 stars being the brightest stars, and magnitude 6 being those that he could barely make out. The magnitude scale is a logarithmic scale, where a difference of 1 magnitude corresponds to a luminosity factor of 2.5. So an $m = 1$ star is about 2.5× brighter than a $m = 2$ star, 6.25× brighter than a $m = 3$ star, and 15.6× brighter than a $m = 4$ star. Equation 1 describes how the magnitudes m_1 and m_2 of two stars depend on the ratio of their brightnesses, b_1 and b_2.

$$m_1 - m_2 = 2.5 \log \left(\frac{b_2}{b_1} \right) \tag{1}$$

Of course, we're talking about *apparent* magnitudes—brightness according to how humans perceive stars from Earth. In reality, stars exist at different distances from Earth and thus how bright something looks isn't a great indication of its power output. This is because stars with very high power outputs can appear dim to us if they're far enough away. Conversely, a star with a low power output can appear to be extremely bright provided it is nearby. To get a more complete picture of a star's brightness, we need to take the distance of a star into account.

The absolute magnitude of a star is the magnitude of a star if it were at a distance of 10 pc (parsecs) away from us, and is denoted by M. You can find the **absolute magnitude** of a star from its apparent magnitude and distance in parsecs with Equation 2.

$$M = m - 5(\log_{10}(d_{pc}) - 1) \tag{2}$$

Here we denote absolute magnitudes with M and apparent magnitudes with m. Of course, as our equipment became more precise and our databases of known stars became more extensive, this sort of classification scheme for brightness became more awkward to use. Since a lower magnitude corresponds to a higher brightness, the magnitude scale suddenly becomes unintuitive now that it isn't just a ranking system (magnitudes can be in decimals). Another problem arises when we want to place nearby planets, the Moon, the sun, and newly discovered bright stars on the magnitude scale. This can mean magnitudes in the negative—a perfectly valid value, but one which can be confusing. Many astronomers today use alternative ways of expressing luminosities, such as expressing them in multiples of the solar luminosity (e.g. Sirius's luminosity is about 25.4 $L_\odot$). Learn more about stellar magnitudes from pages 595–599 from Fraknoi et al. (2018).

2.1 Standard photometric systems

Okay, so, we've talked about apparent and absolute magnitudes as a corollary to a star's luminosity. Earlier, we defined luminosity to be the total energy output across all wavelengths of light. This is fine to talk about in theory, but in practice we usually talk about luminosity for a certain band of wavelengths. It's kind of impractical to make measurements across most of the electromagnetic spectrum just to talk about how bright something is. Also, stellar luminosities vary across wavelengths, and we can talk about a lot more astronomy with different standardised bands.

There are a couple of standardised bands that astronomers use usually. One of these photometric systems is the **UBVRI** system, also known as the **Johnson-Cousins** photometric system. Each letter in 'UBRVI' corresponds to a specific wavelength range, as highlighted by Figure 1. Newer systems like the Sloan **ugriz** photometric system are becoming popular as well.

So in summary, when we use magnitudes, we'd probably be specifying the band alongside it. For example, m_{V} is the apparent magnitude in the V band while M_{B} is the absolute magnitude in the B band.

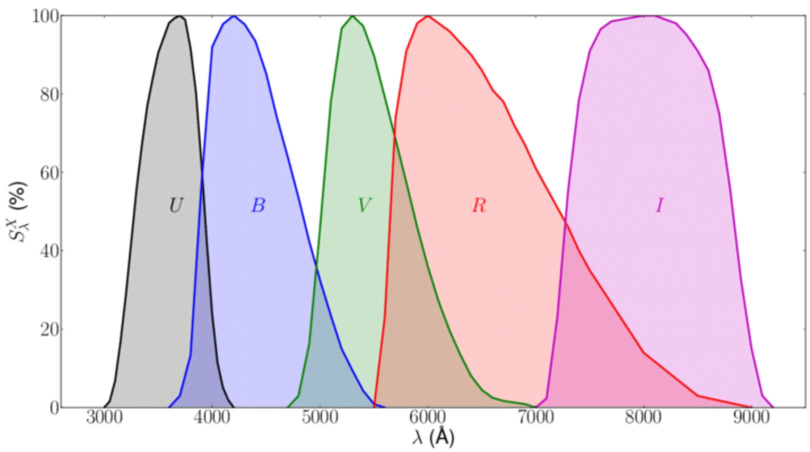


Figure 1. The UBVRI bands their approximate colours. Credits: Coelho, Rodrigo & Calvão, Mauricio & Reis, Ribamar & Siffert, Beatriz. (2014). Standardization of type Ia supernovae. European Journal of Physics. 36. 015007. 10.1088/0143-0807/36/1/015007.

2.2 Colour Indices

Now that we're familiar with the UBVRI colour bands, we can explore colour indices. If you subtract a magnitude in one colour band from the magnitude in another, you get a colour index, say (B-V). Since M_{B} is the magnitude in the 'bluer' band while M_{V} is the magnitude in the 'redder' band, (B-V) can then tell us how blue or red a star is. A smaller (B-V) means a blue-er star, while a larger (B-V) means a redder one.

A colour index also lets us estimate the temperature of a star's photosphere (or T_{eff}). This is because the continuous radiation of a star is well-modelled by a blackbody with the same temperature as the star's photosphere. Like a blackbody, the emission from a star has a *peak wavelength* (λ_{peak}), or the wavelength of light at which it emit the most radiation. λ_{peak} changes with the photosphere's temperature as described by **Wien's displacement law** in Equation 3.

$$\lambda_{peak} \sim \frac{3 \times 10^{-3}}{T_{\mathrm{eff}}} \tag{3}$$

Thus, hotter stars have a smaller λ_{peak} and are bluer, while cooler stars have a larger λ_{peak} and are redder. Learn more about the colour-temperature relationship and colour indices on pages 600–602 from Fraknoi et al. (2018).

3 The H-R Diagram

In the early 20th century, two astronomers independently plotted stars' absolute visual magnitude (magnitude within the visual spectrum) against their spectral type. The Danish Ejnar Hertzprung and the American Henry Norris Russell found a clear correlation between a star's magnitude and spectral type. This kind of diagram is now known as a Hertzprung-Russell or **H-R diagram**. Figure 2 shows Henry Russell's original H-R diagram.

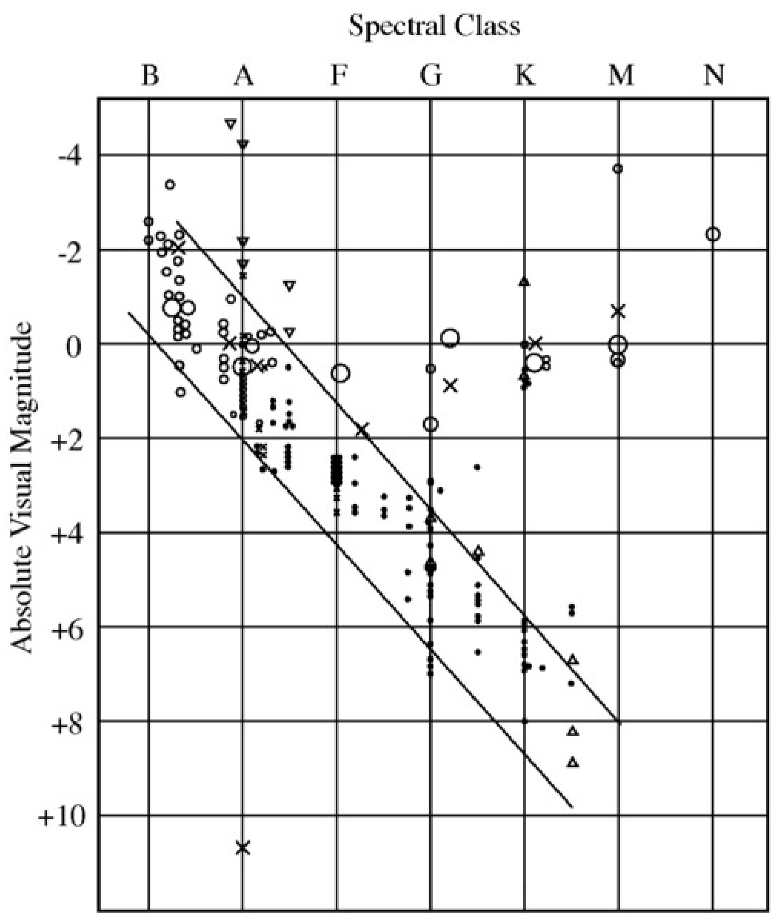


Figure 2. Henry Russell's original H-R diagram with absolute visual magnitude on the vertical axis, and spectral type on the top-vertical axis. Credits: Henry Norris Russell, *Relations between the spectra and other characteristics of stars*, Popular Astronomy 22, 275-294 (1914).

3.1 Why that pattern?

The two astronomers observed that most stars seemed to reside on a linear region stretching from the top-left to the bottom right of the H-R diagram. Additionally, we know that the T_{eff} of a star decreases down the OBAFGKM classification scheme. Thus, the brightest and hottest stars seemed to reside on the top-left, while the dimmest and coolest tended towards the bottom-right. This is also why we get the same pattern when we use (B-V) on the x-axis instead of spectral class or T_{eff}—the colour of a star can tell us its temperature. Figure 3 shows us an H-R diagram with spectral class, temperature, and colour. Figure 4 shows an H-R diagram with stars within 100 pc of the Sun, made with measurements from the Hipparcos satellite.

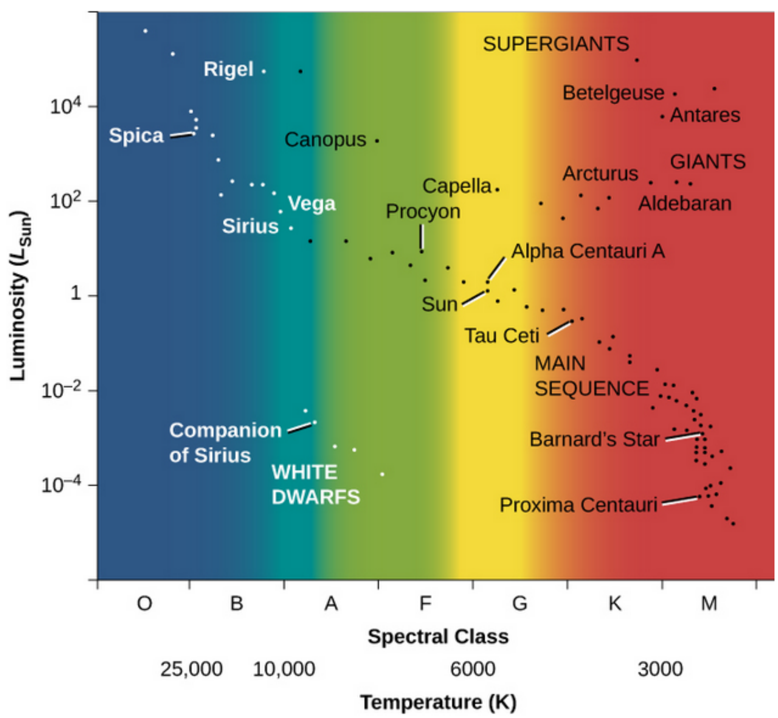


Figure 3. An H-R diagram displaying how stellar colour, spectral type, and temperature correlate and may be used on the x-axis. Credits: *Andrew Fraknoi et. al. OpenStax Astronomy.*

[Q1] I get a set of random stars from all over the Milky Way, and plot an H-R diagram with the (B-V) colour index on the x-axis, and apparent V magnitude on the y-axis. I plot them out and obtain Figure 5. Why don't I see the nice, familiar H-R pattern?

3.2 What do H-R Diagrams Say?

H-R diagrams can tell us a lot about the stars they include. The most prominent feature of the H-R diagram is undoubtedly the approximately linear pattern found from the top-left (hot, bright stars) to the

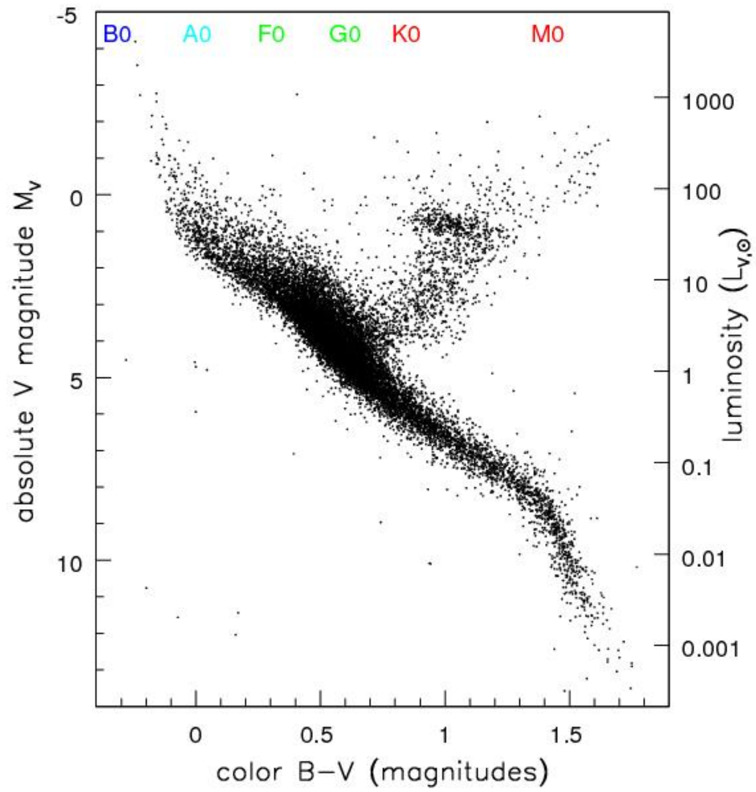


Figure 4. H-R diagram of stars within 100 pc with measurements from the Hipparcos satellite.

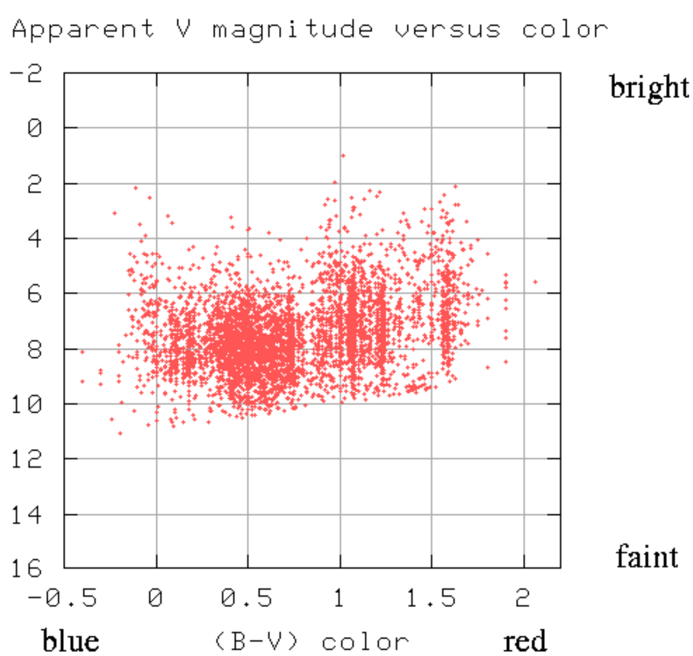


Figure 5. An H-R diagram for a set of stars from different places in the Milky Way. Quite random! Credits: *Michael Richmond, Rochester Institute of Technology.*

bottom-right (cool, dim stars). This region is called the **main sequence**, and the vast majority of stars ($\sim 90\%$) reside here. One might ask, what determines where on the main sequence a star resides? A star's position on the main sequence generally depends on its mass. That kind of makes intuitive sense. More massive stars have their cores under greater gravitational pressure, leading to faster fusion, higher energy outputs, and greater temperatures. Less massive stars contend with less gravitational pressure, and hence burn their fuel slower.

But an H-R diagram can show more than just the main sequence. Figure 4 clearly shows a significant population of stars branching out from the main sequence towards the upper-right corner. These stars are **red sub-giants**, **giants**, **super-giants**, and **hyper-giants**. When a main sequence star depletes its hydrogen reserves, it expands, and begins to shift upwards and to the right of the H-R diagram. This means that the star's T_{eff} lowers while it simultaneously increases in luminosity.

[Q2] Why do you think a typical main-sequence star, as it transitions into a red star, increases in luminosity even though its T_{eff} decreases?

The H-R diagram shows us several more populations of stars residing in different locations due to their unique properties. Figure 6 shows several such locations.

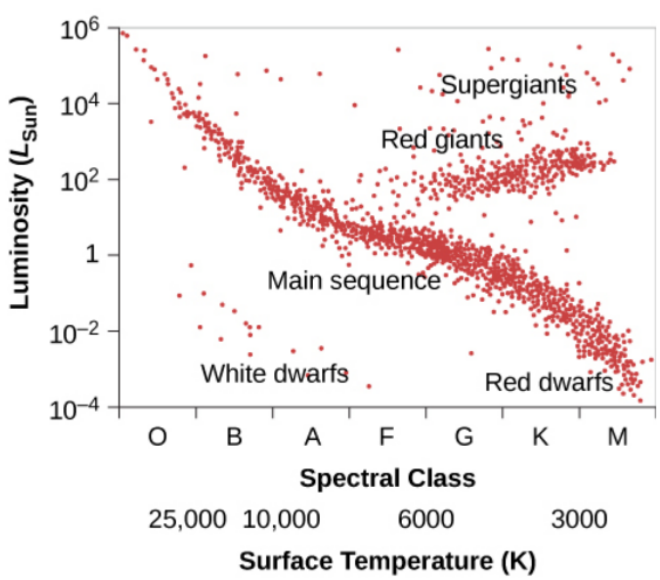


Figure 6. An H-R diagram displaying where different stellar populations reside in the H-R diagram. Credits: *Andrew Fraknoi et. al. OpenStax Astronomy.*

4 Clusters

What are star clusters? Believe it or not, astronomers aren't certain! They tend to have several definitions depending on what they're doing. Star clusters (not to be confused with galactic clusters) can be broadly thought of a collection of stars born from the same gas cloud around the same time, bound together by gravity. Additionally, all stars in a cluster are near each other, and so their approximate distance to us is pretty much the same.

Clusters have historically been separated into open clusters (generally found within the disk of our galaxy) and globular clusters (usually residing in the galaxy halo). Open clusters are often young ($\sim 10^7$ yr), loosely grouped, and generally contain a few hundred members. Globular clusters, on the other hand, are generally older ($\sim 10^{10}$ yr) and quite dense, with tens of thousands to millions of stars packed within a ~ 10 light-year radius. Figure 7 shows some examples.

Figure 7. The Pleiades (l) is an open cluster about 400 light-years from earth and contains about 3,000 stars. The Hercules cluster (right) is a globular cluster about 22,000 light-years away and contains as many as a million stars. Image Credits: NASA, ESA, AURA/Caltech, Palomar Observatory (*Pleiades*), Sid Leach/Adam Block/Mount Lemmon SkyCenter *Hercules*.

4.1 Cluster H-R diagrams

[Q3] In [Q1], I got a set of random stars from all over the Milky Way, and plotted an H-R diagram. This resulted in no recognisable pattern (Figure 5). Now, I've gotten a set of stars which are from the same open cluster, and plotted an H-R diagram with the (B-V) colour index on the x-axis, and m_V on the y-axis. This gives me Figure 8. There's that pattern again! Why do I see it now?

[Q4] Say you had photometric data for an open cluster. Imagine that you make two H-R diagrams from that data: one with m_V on the y-axis and (B-V) on the x-axis, and one with M_V on the y-axis and (B-V) on

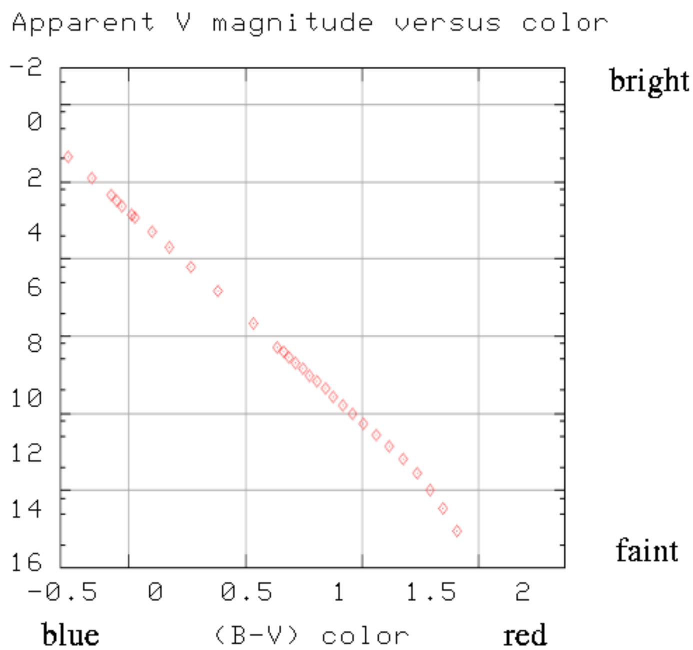


Figure 8. An H-R diagram for a set of stars from the same cluster. Nice pattern! Credits: *Michael Richmond, Rochester Institute of Technology.*

the x-axis. What difference would you see? You can draw a rough diagram on your notebook to highlight the difference that you'd see. It's okay if you're not 100% sure!

[Q5] You're about to check the hypothesis you made in [Q4]. If you haven't opened the Google Colab notebook associated with this manual yet, this is a good time to do that. (It can be found in this lab's webpage). You have photometric data from **NGC 2168**, also known as **Messier 35**. M35 is an open cluster some 2800 ly away from us, west of Gemini (see Figure 9). You're going to plot one m_V vs. (B-V) diagram. Then, plot of M_V vs. (B-V) (don't worry, there's detailed instructions in the notebook!). Was your hypothesis correct? If it was, confirm the reason. It it wasn't, try and explain what was wrong with your conjecture.

[Q6] Now that you've plotted m_V vs. (B-V) and M_V vs. (B-V) for M35, find out how far away it is in light-years on your Colab notebooks.

4.2 ZAMS

What would we see if we got a bunch of stars in a cluster which have just been born, and plotted them on an H-R diagram? Figure 10 shows such an H-R diagram. It's pretty much linear and we don't that branch

Figure 9. Messier 35 in all its glory. Its stars are spread accross 11 ly. Credits: *Wikisky*.

of stars moving away from the main sequence. Astronomers call this sort of H-R diagram the ZAMS (zero-age main sequence), which is an H-R diagram with every star in the main sequence. ZAMS are quite rare in reality, but we do see some ZAMS-like behaviours in some stellar populations.

[Q7] Why do we not see that branch of stars moving away from the main-sequence in a young cluster?

[Q8] Why could ZAMS be so rare?

4.3 The Shapes of Cluster H-R Diagrams

We've discussed previously how H-R diagrams may show stars in different locations depending on their properties and on what stage of their evolution they might be on. Often, we are able to see the main-sequence (well defined from the top-left to the bottom-right), and a branch of red stars moving away from the main-sequence to the upper-right side of the diagram. Figure 4 shows this quite clearly, with red star branch seemingly 'protruding' from the continuous main-sequence. The H-R diagram for a cluster, however, can look somewhat different. Figure 11 shows the H-R diagram for the globular cluster 47 Tucanae. Try to note some differences when compared with Figure 4, the H-R diagram from the Hipparcos

Figure 10. H-R diagram for a very new cluster. Credits: *Michael Richmond, Rochester Institute of Technology.*

satellite.

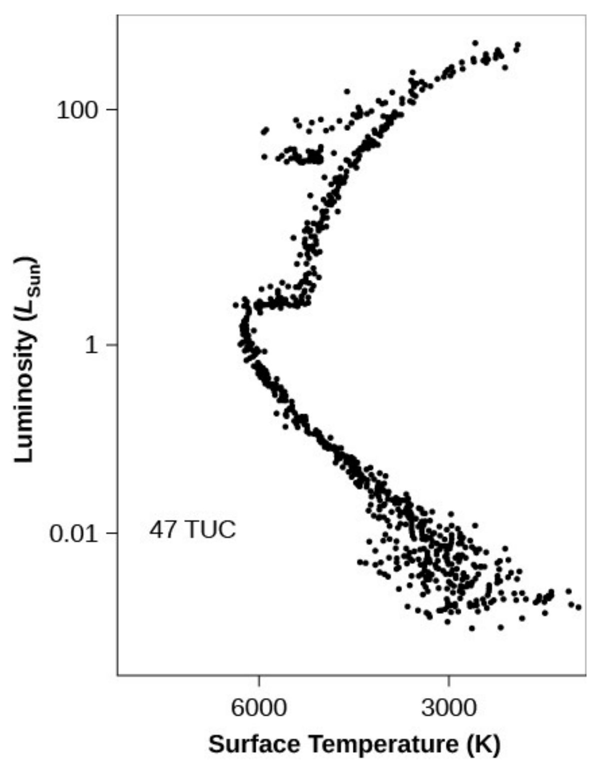


Figure 11. H-R diagram for the Cluster 47 Tucanae. Compare this to Figure 4, which displays the H-R diagram of stars that aren't all in one cluster. Credits: *Andrew Fraknoi et. al. OpenStax Astronomy.*

I'm sure you'll have found many differences. One of the more prominent ones is that the main-sequence does not seem to extend all the way to the top-left, where the brightest and hottest stars usually are.

Additionally, this end is where the red star branch seems to originate from. Incidentally, the point where the red-giant branch meets the main-sequence is called the **main sequence turn-off point**.

4.4 The Time Evolution of Cluster Stars

Figure 12 shows the H-R diagrams for several different clusters. Among these is NGC 2362, which (approximately) displays the legendary ZAMS behaviour. Other clusters can also be seen with a very interesting variation—different clusters seem to have different main-sequence turn-off points.

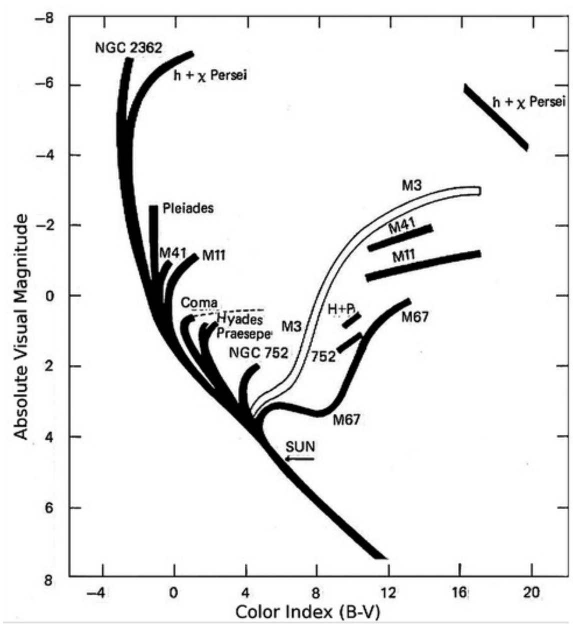
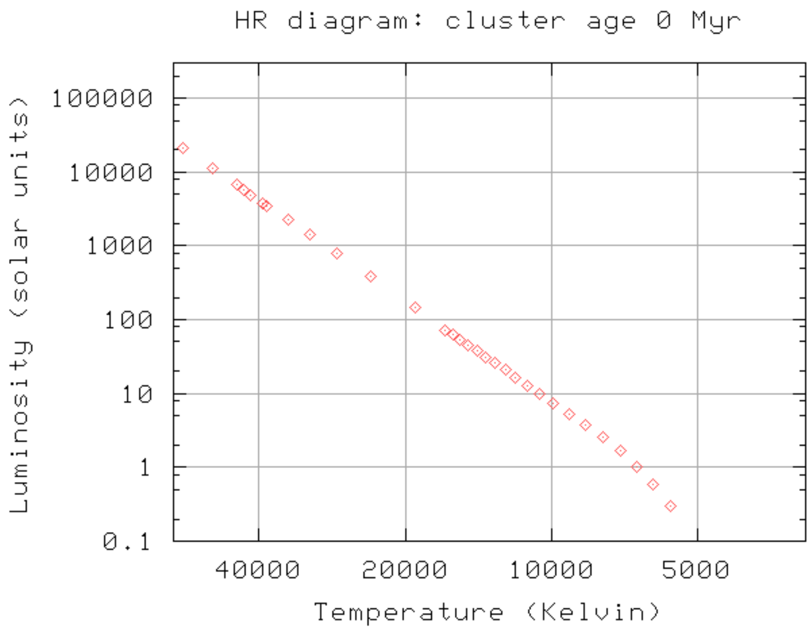


Figure 12. Several clusters being plotted on an H-R diagram. We can see that different clusters start to move away from the main sequence at different points. Credits: Alan Sandage *Astrophysical Journal 126, 326*, (1957).

Why could this be? Professor Michael Richmond's excellent GIF, which shows a cluster's H-R diagram as it ages, might be helpful in thinking about this. (Open this file in a dedicated PDF program to make sure the GIF works.)



Play/Pause

[Q9] With these media, could you explain the apparent differences in the H-R diagram of a cluster and that of a random set of stars? (Hint: remember the aforementioned basic facts about cluster ages, and what determines how fast a star will turn into a red star.)

[Q10] If you had a function to determine the lifespan of a main-sequence star, how would you find the age of an open cluster?

5 Isochrones

A ZAMS can show us how a cluster might look on an H-R diagram when it is young. However, it can't tell us how a cluster may look when it is at different ages, and we might be really interested in knowing that as many clusters are at different points in their evolution. It'd be nice to know how a cluster might look at different ages. That's where isochrones come in. Astronomers can create H-R diagrams for how clusters may evolve through time by using theoretical models taking into account a cluster's population, kinematics, metallicities, dust extinction, and atmospheric models. Many of the professional models astronomers use are complex and computationally expensive to compute. That's why a lot of these available pre-processed on dedicated astronomical servers! A popular stellar evolution and population synthesis code is Modules for Experiments in Stellar Astrophysics, or MESA. MESA documentation may be found at `https://docs.mesastar.org`. From this code base, a Harvard team has created MESA Isochrones & Stellar Tracks, or MIST (`https://waps.cfa.harvard.edu/MIST/`), dedicated to creating and hosting isochrones and stellar evolutionary tracks for populations with a wide range of properties.

[Q11] MIST isochrones and tracks are widely used by professional astronomers to calculate the age of star clusters. In fact, please turn to the Colab notebook accompanying this manual to find out the age of your own cluster!

Acknowledgements

This lab was created with media and images from a wide range of sources. In particular, Michael Richmond's *Interpreting the H-R diagram of stellar clusters* (licensed under Creative Commons CC BY-NC-SA 2.0) provided informative graphics. The OpenStax Astronomy textbook (Andrew Fraknoi, David Morrison, Sidney C. Wolff et. al.) provided diagrams and is referred for further reading. Stellar isochrones from MIST were utilised in this lab. Photometric data for NGC 752 was obtained from the *Photometry and radial velocities in NGC752 (Daniel+, 1994)* data collection on VizieR. Photometric data for M53 was obtained from the *BV photometry of M53 (Rey+, 1998)* data collection on VizieR. This document was built upon the Royal Society Open Science Overleaf template, which is licensed under Creative Commons CC BY 4.0.

References

Angela Adamo, Peter Zeidler, J. M. Diederik Kruijssen, Mélanie Chevance, Mark Gieles, Daniela Calzetti, Corinne Charbonnel, Hans Zinnecker, and Martin G. H. Krause. Star Clusters Near and Far; Tracing Star Formation Across Cosmic Time. , 216(4):69, June 2020. doi: 10.1007/s11214-020-00690-x.

Jieun Choi, Aaron Dotter, Charlie Conroy, Matteo Cantiello, Bill Paxton, and Benjamin D. Johnson. Mesa Isochrones and Stellar Tracks (MIST). I. Solar-scaled Models. , 823(2):102, June 2016. doi: 10.3847/0004-637X/823/2/102.

Scott A. Daniel, David W. Latham, Robert D. Mathieu, and Bruce A. Twarog. A Photometric and Radial-Velocity Analysis of the Intermediate-Age Open Cluster NGC 752. , 106:281, March 1994. doi: 10.1086/133379.

Aaron Dotter. MESA Isochrones and Stellar Tracks (MIST) 0: Methods for the Construction of Stellar Isochrones. , 222(1):8, January 2016. doi: 10.3847/0067-0049/222/1/8.

Andrew Fraknoi, David Morrison, and Sidney C. Wolff. *Astronomy*. OpenStax, 2018.

Bill Paxton, Lars Bildsten, Aaron Dotter, Falk Herwig, Pierre Lesaffre, and Frank Timmes. Modules for

Experiments in Stellar Astrophysics (MESA). , 192(1):3, January 2011. doi: 10.1088/0067-0049/192/1/3.

Bill Paxton, Matteo Cantiello, Phil Arras, Lars Bildsten, Edward F. Brown, Aaron Dotter, Christopher Mankovich, M. H. Montgomery, Dennis Stello, F. X. Timmes, and Richard Townsend. Modules for Experiments in Stellar Astrophysics (MESA): Planets, Oscillations, Rotation, and Massive Stars. , 208 (1):4, September 2013. doi: 10.1088/0067-0049/208/1/4.

Bill Paxton, Pablo Marchant, Josiah Schwab, Evan B. Bauer, Lars Bildsten, Matteo Cantiello, Luc Dessart, R. Farmer, H. Hu, N. Langer, R. H. D. Townsend, Dean M. Townsley, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Binaries, Pulsations, and Explosions. , 220(1):15, September 2015. doi: 10.1088/0067-0049/220/1/15.

Bill Paxton, Josiah Schwab, Evan B. Bauer, Lars Bildsten, Sergei Blinnikov, Paul Duffell, R. Farmer, Jared A. Goldberg, Pablo Marchant, Elena Sorokina, Anne Thoul, Richard H. D. Townsend, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Convective Boundaries, Element Diffusion, and Massive Star Explosions. , 234(2):34, February 2018. doi: 10.3847/1538-4365/aaa5a8.

Soo-Chang Rey, Young-Wook Lee, Yong-Ik Byun, and Mun-Suk Chun. Ccd photometry of the globular cluster m53. i. color-magnitude data and blue straggler stars. *The Astronomical Journal*, 116(4):1775, oct 1998. doi: 10.1086/300555. URL https://dx.doi.org/10.1086/300555.

Barbara Sue Ryden and Bradley M Peterson. *Foundations of Astrophysics*. Addison-Wesley, 2011.